Two Implications and the Dual-Process Theories of Reasoning

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Abstract. System 1 of fast, intuitive, associative, and effortless reasoning is shown to be just as logical as System 2, which in the dual-process theories of reasoning is said to draw consequences in rule-based, rational and criticised fashions: the difference is only that the former draws conclusions in a diagrammatic, positive and implicational fragment of logic. This fundamental logical connection between the two systems is then applied to explain away cognitive biases in the card selection task, which is thus shown to cease to represent a paradigm case of confirmation bias: both systems of reasoning exhibit processes of logical inferences.

Keywords: Logic; Cognition; Dual-processes; Diagrammatic logic; Peirce's Graphs; Positive Implication; Wason card selection task.

1 Introduction

Ability to reason is one of the great unanswered questions in the evolution of the genus *homo*. Contemporary studies in the cognitive science of reasoning have mostly been geared towards psychology, formal logic having a supportive function, if any. Even the mental model theories [9, 10, 23] that admit non-monotonic logics assume that reasoning ultimately depends not on logical forms but on mental models of situations. While this assumption was criticized among others in Hintikka (1986), it did not leverage the supportive function of logic in cognitive and psychological studies.

Theories of reasoning can generally be categorised under two headings: the *dual-process theories* [3, 11, 21, 26] and the *single-system conceptions* [4, 12, 15, 22]. We argue that a logical perspective is indispensable in dual-process theories but do not criticise psychological conceptions from the points of view of logic. Neither 'logical psychologism' nor 'psychological logicism' are fruitful stances. Psychology studies human thought and action in actual and prospective cases while logic is in the business of analysing the general conceptions of these actions.

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We stress that getting the meaning of these central concepts right is essential to the understanding of reasoning in its multiple forms. Basic principles of human reasoning need to the examined and their logical content demonstrated. The conception of logic used deals not with various matters of *human thinking* but rather is used to scrutinise how one thought (proposition, assertion, content) ought to be related to another. Only if these connections take the form of premises-and-conclusion in terms of the relation of consequence, we can say that reasoning appears. Only if these connections agree with specific rules or principles of reasoning, would any piece of reasoning possess some desirable properties, such as being valid, secure, reliable, fruitful, or evident to reason. And only if we can bring out what these basic rules or principles of reasoning are, can the value of those interlinking processes be adequately evaluated. We agree with the position that people are not irrational but logical—if people "ignored logic all the time, extracting the wrong information from the data at their disposal, it is hard to see how our species could survive", as the rules of task-dependent forms of inference "include more heuristic 'default rules' that are not valid in our strict sense" [27, p. 2].

This paper suggests another solution. A possibility for a fruitful cooperation between logical and psychological theories of reasoning springs from Charles Peirce's theory of reasoning, in particular the theory of *existential graphs* (EG; [1, 8, 20, 28]). EG makes explicit the logical power of reasoning in diagrammatic form. Its results can, we propose, solve prevailing issues in cognitive psychology concerning the Wason card selection task, dispensing with cognitive-bias explanations associated to the results of those experiments. We apply Peirce's philosophy of logic and reasoning to those results.

2 Two Systems of Reasoning and Two Types of Scrolls

Dual-process or dual-system conceptions have a prominent role in contemporary studies of human reasoning. Its alternative is commonly bundled as 'single-system' conceptions [4, 12, 15, 22]. Dual-process theories date back to William James [7] but rose to prominence belatedly. From the works including [3, 21, 26], the idea was broadened to the decision-making theory by [11], among others.

Dual-process theories contrast *spontaneous* and *deliberate* reasoning. Theories vary in details but share the common core. The two types are taken to exploit what is termed in cognitive sciences as Systems 1 and 2 (S1, S2). S1 is responsible for performing fast, intuitive, associative and effortless reasoning, whereas S2 produces rule-based, rational and criticised consequences of our thoughts with increased cognitive effort and time expended on tracing those consequences than what the S1 would do.

It is also commonly maintained that in ordinary circumstances, subjects appeal to the first type as it is easier to use and leads to effective solutions and short-cuts much needed in actual reasoning cases. It is only when S1 fails us (which would happen, it is said, in non-typical situations), S2 may interject its correctives to the subject's performance of reasoning.

The evidence for the presence of the two systems of cognition has been drawn from numerous experimental results, most famously the Wason selection task [29, 30]:

– A subject sees four cards. Two cards have their letter sides up; the other two the have their number faces up. It is common knowledge that each card has a letter on the one side and a number on the other. Participants are then asked to answer the question: Which cards they should turn over to prove the rule “If on one side of the card there is an E, then on the other side there is a 2”.

The experiment is built on the schemata of Modus Ponens and Modus Tollens. “If an E, then a 2” does not mean that only an E is paired with this number. The results have been interpreted to show that a vast majority of people ignored the ‘not only’ condition. (The correct answer is ‘E’ and ‘7’.)

A negative version of this selection task has indeed been taken to reinforce the standard lesson:

– What would happen when the rule “If on one side of the card there is an E, then on the other side there is a 2” is modified into “If on one side of the card there is an E, then on the other side there is *not* a 2”, while rest of the conditions are left unaltered?

The answer is that modified experiments suggest that those participants who are asked to prove the latter conditional usually do significantly better in producing the correct answer.

Proponents of the dual-process theories have concluded that people do not, in general, utilise logical reasoning to resolve such tasks. Rather, subjects appeal to what is provided by the fast track of S1. Logic can justify or imitate reasoning, especially when it comes to its non-monotonic forms, but the theory of logic itself is held incapable of assisting us in producing conclusive or even partially satisfactory answers to the question of what constitutes reasoning.

Indeed the logical structure of the two-systems dichotomy is not clear. Exactly where does the first cease to apply and when does the second system take over? Or are the two systems rather located at the ends of a continuum, say from analogical to digital processing? Admitting that S2 corrects the fast-and-frugal mistakes made by S1 does not alone mean that S2 is free from such errors. People notoriously repeat the same classes of fallacies yet expect different outcomes. Why would S1 not use logical reasoning, then?

Here we find support for the possibility that both systems may well rely on the same general logical schemata of reasoning. It is just that humans may fail to observe and identify the relational structure between them.

3 Logical Graphs for Cognition

The correlation between logic and cognitive processes (or computations) may be described as:

[Logic] can never be the whole story of the implementation level (which by definition involves the physical instantiation of an algorithm). Nevertheless, logic can help bridge the gap between the implementation of an algorithmic level by analyzing structural similarities across different proposed instantiations. . . . Logical analysis can distinguish the commonalities across implementation-level hypotheses form their true disagreements. [6, p. 796]

Arguably this confirms that the turn to logic is a tricky one; contemporary logic professes to be formal, but the formality itself is a Janus-faced entity: technical details can overshadow long-lasting visions on its true subject matter.

It is far from clear whether computational models simulate or replicate the causal powers of situated reasoning tasks. Recently, a credible framework has been proposed "for epistemic logic, modeling the logical aspects of System 1 ('fast') and System 2 ('slow') cognitive processes, as per dual process theories of reasoning" . . . It is applied to three instances of limited rationality, widely discussed in cognitive psychology: Stereotypical Thinking, the Framing Effect, and the Anchoring Effect" [24]. The model imitates the duality of 'moderately rational agents', but this duality itself is taken for granted in the above work, and the methodological and philosophical side of the question is swept aside.

This solution is nevertheless along the lines of ours that reasoning is altogether logical. At the same time, we claim that logic can do more than usually thought. It can be applied to cases that exercise reasoning in the course of empirical investigation. But in order to do that, we need another approach; namely, we need a theory in which philosophical, empirical and formal parts are better balanced. We now show that EG can assume this role.

EG is a diagrammatical logical system that includes several further theories, such as the Alpha, the Beta, and the Gamma parts, which roughly correspond to propositional logic, first-order logic and modal logic, respectively [13, 17, 20, 28]. Its basic units are graphs, as illustrated in Fig. 1.

Fig. 1.

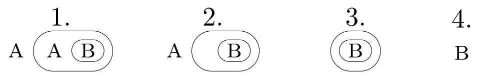
Graphs are propositional expressions "of any possible state of the inverse" (CP 4.395). Technically, a graph signifies a "general type of whatever means the same thing and expresses that meaning in the same way . . . ; while that which is scribed once and only once and embodies the graph" is a graph-instance (R 45); graphs actually presented or *scribed* on the *sheet of assertion.* Unlike mathematics, logic is not abstract science.

We confine our presentation to the Alpha part that agrees with propositional logic (the two-element Boolean algebra). Its syntactical part is presented with the sheet of assertion (which represents tautology) and the components of graphs, called the *cuts* (which are non-overlapping closed curves representing scope and negation). Two graphs on the same sheet are said to be *juxtaposed* (Fig. 1 on the left). The sheet itself is a proposition expressing tautology while juxtaposition corresponds to conjunction. Under the standard interpretation, the cut represents Boolean complementation. In terms of the Alpha part, the graphs (Fig. 1) should be read as $(A \wedge B)$ or $(B \wedge A)$ and $\overline{(A \wedge \bar{B})}$ or $(A \rightarrow B)$ respectively.

This part of the theory presumes the presence of the set of (sound and complete) transformation rules that defines reasoning as a series of insertions and omissions. Any graph may be *inserted on any oddly enclosed area* or be *erased whenever evenly enclosed*. Any graph on any area may be scribed on the same or any other area contained within it. In case a graph results from such copy-paste *iterations*, it may be erased (*de-iteration*). If nothing else than a blank occupies an area between two cuts, this *double cut may be removed*. Also, a double cut may always be *added around any graph*.

The graph transformations of Fig. 2 (Modus ponens) demonstrate how these rules may be used.

Fig. 2.



EG is a philosophically and formally balanced system of logic. It is a logical system, yet can also be positioned as a *cognitive resource* (Pietarinen 2011) that provides "a rough and generalized diagram of the Mind", one which "gives a better idea of what the mind is, from the point of view of logic, than could be conveyed by any abstract account of it" (R 490; [19, p. 900]).

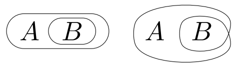
Diagrammatic transformations show how one graph is turned into another one along logical consequence relation. In other words, the theory takes graphs as "moving-pictures of thoughts" and these 'moving pictures', in turn, illustrate the core that governs reasoning. Peirce calls it leading or guiding principle of reasoning:

> That which determines us, from given premisses, to draw one inference rather than another, is some habit of mind, whether it be constitutional or acquired. The habit is good or otherwise, according as it produces true conclusions from true premisses or not; and an inference is regarded as valid or not, without reference to the truth or falsity of its conclusion specially, but according as the habit which determines it is such as to produce true conclusions in general or not. The particular habit of mind which governs this or that inference may be formulated in a proposition whose truth depends on the validity of the inferences which the habit determines; and such a formula is called a *guiding principle* of inference. (CP 5.367)

The guiding principle constrains the ways in which information flows and conclusions determined, secured and supported by their premises. The leading principle governs all reasoning in general and is not limited to deductive reasoning.

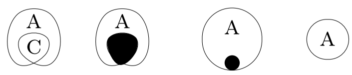
4 Guiding Principle, Scroll and Two Implications

The guiding principle is intrinsically connected to the interpretation of implication. In EG, it is presented either as two nested cuts or as "the scroll" composed of one continuous line or two closed lines one inside the other (Fig. 3). In scrolls, the antecedent is placed in the outer compartment while the consequent is situated in the inner compartment. The scroll correspondence to the idea of leading principle, namely that implication is the basic logical sign. The strategy of proving logical laws illustrate this since any proof starts with an insertion of a double cut on the blank sheet of assertions.

Fig. 3 

However, the scroll does more than this. It also introduces negation by generating the cut (Fig. 4). The cut is the result of evolution from the scroll. That process is in the very operation of logical graphs: "A certain development of reasoning was possible before . . . the concept of falsity had ever been framed" (R 669, 1910; [19, p. 920]). Such a state of affairs can following Peirce be called *paradisiacal* (R 669). Under it, our assertions take the form "If X be true, then every assertion is true." At the same time, for those living under such conditions, "it will soon be recognized that not every assertion is true; and that once recognized . . . , one at once rejects the antecedent that lead to that absurd consequence" (*ibid.*).

Fig. 4 graphically reproduces this process: we move from "If A is true, C is true" via "If A be true whatever can be asserted is true" and "A is not true and the inner close being cut very small" towards the idea of negation. The black spot that we see in the second and third graphs is the "blot". The color or shade means that its area is fully occupied and no proposition could be added to it, not event the blank. The blot contrasts with the sheet but is positive and affirmative rather than negative assertion or denial. As the size of that area does not change the cardinality of truths included in it, the inner loop with the blot in it could just as well be atrophied until the loop coincides with the boundary of the outer circle of the scroll at their intersection point:

Fig. 4 (a, b, c, d respectively) 

Such process of evolution of a fundamental logical conception now results in two kinds of implications: *paradisiacal* implication that lacks the conception of a negation, and logical or *material* implication [14].

This split marks the connection with contemporary theories on reasoning. Both implications have the same structure and both illustrate the workings of guiding principle. But both have different presuppositions. Paradisiacal implication is the primitive of the language whereas the material one corresponds to the conditional *de inesse* as studied in standard logic.

This split at the level of logic can be confirmed by data. Humans—as manifested in such examples as the development of grammar in *homo erectus*, infants below c.14 months of age, as well as most other primates—seem unable to conceptualise the ideas of the negation, contradiction, or even that of absence. Difficulties show up at the levels of grammar acquisition and in the production and comprehension of illocutionary forces involved in them (see e.g. [2]). Yet reasoning is present, admirably carried out in terms of positive instances only. But when crucial word–object relations break down and the subject becomes conscious of it, hypothetical conditionals emerge. With the emergence of hypotheticals, other concepts such as axioms follow suit, indicating that there was a hidden presence of falsity all along.

5 Wason's Card Selection Task and Two Implications

How does this relate to what was previously said? Let us go back to the proposed solution to the selection task and to the claim that S1 need no appeal to logical reasoning. This conclusion is too hasty. For all reasoning has to have certain rational foundations. Although questions of rationality and sound reasoning (as opposed to the reasoning that is, say, bad, invalid, unsound, or unethical) may be treated quite differently in different domains and applications, they have to be predicated on certain normative powers of logic that can cater for rationality.

Recall that reasoning takes place as soon as there are norms that allow composition of thoughts in terms of premise–conclusion consequence relation. Likewise, one must acknowledge that reasoning is not taking place when a 'therefore', in whatever form, is absent. This demarcation between the thoughts that follow from other thoughts in the premise–conclusion sequence and those that do not is significant, since premises are the material that ought to support their conclusions (they are *signs* that represent their conclusions as their objects). Otherwise, the whole edifice of sound reasoning would recede from view.

Let us take the scroll to provide premise–conclusion schemata and let us link its paradisiacal and material versions to S1 and S2, respectively. When participants choose their cards, they tend to admit that their reasoning is grounded on what they see, and for such reasons do not proceed thinking of alternative solutions. But if so, the selection task is not a textbook case of confirmation bias: the subject's ignorance of a relevant piece of information is not a bias but a *crucial part of logical reasoning at the level of the paradisiacal implication*. The results of the selection task can be interpreted to mean that people are prone reasoning at the level of the scroll that is paradisiacal, an implication devoid of the conception of negation or falsity.

Indeed paradisiacal illation is a natural, even primitive, operation of thought. It presumes that reasoning can proceed even when there is no negation present. What this confirms is not a confirmation bias but the fact that the processes of refutation are less natural to be exhibited in human reason than confirmation.

In other, Peirce's, words, the blot is an affirmative constant, and "affirmation is psychically the simpler" (R L 386). Negation, in contrast, is a *polar* phenomenon, and for negation to make itself manifest, certain further conceptions of *boundaries* are needed first to be maintained in one's mental images of diagrammatic thought. Geometrically, at least two areas are simultaneously present, namely something that exists and something that exhibits mere possibility. This is not a trivial task to be accomplished, and it is no major wonder that S1 tends to take precedence. In short, in human tasks of the sort the selection tests study we see a spontaneous creation of contexts for *positive fragments of logic*, fragments devoid of the conception of negation.

The benefit of this way of looking at the interpretation of the results of the selection experiments is that when the negative element is added to the instruction of the task, participants need not move from one vocabulary or mental notation into another. The rule "If on the one side of the card there is an E, then on the other side there is not a 2", readily includes a negative concept "not a 2".

This setting differs markedly in meaning from the positive token of the standard version of the test. However, in both cases, subjects can imagine the same sign of implication; in our terms, the same diagrammatic form of the scroll may be in operation. But in this latter task, however, what triggers the higher success rate is that subjects also recognize that "not every assertion is true". This triggers the aforementioned process in which implication evolve into a negation and sets the latter as an explicit element of the vocabulary of one's mental representation or language in which the respective inferences are enacted.

Last, we can note that the two meanings of the implication, or the evolution of negation from the scrolls, can also explain the single-process approaches to reasoning. An explanation of the Wason selection task offers a vivid illustration of it: "Affirmative rule makes no prediction on the letter to be found on the hidden side of the 2 card, but the negative version of the rule does: an E on the hidden side of the 2 card would falsify the negated rule" [22, p. 43]). But why does the negative version form such a prediction? This further but essential question remains unanswered under their approach.

According to the relevance-theoretic account the improved performance is wholly determined by the heightened expectation of relevance, in a predictable way, by the content and context of the rule: "[B]y manipulating subjects' expectations of relevance, correct performance can be elicited in any conceptual domain . . . Relevance Theory has been initially developed on the basis of philosophical arguments, general psychological considerations, and empirical work in linguistics", and the selection task does not "reveal anything profound about reasoning proper" [25, p. 89]. Our hypothesis, in contrast, offers another explanation: the affirmative rule does not make it evident that 'not every assertion is true' while the negative rule does.

6 Conclusion

The proposed analysis of the Wason selection task is only a case by which we can evaluate the two Systems dichotomy anew. S1 can be treated just as logically as S2, with a diagrammatic positive implicational fragment of Peirce's logic of EG responsible for its operation according to the leading principle. The principle works at both deductive and defeasible levels. Such renewed logical foundations for cognition can not only explain away cognitive biases (and the card selection task, for instance, cease to be a paradigm case of confirmation bias) but grounds both systems of reasoning in logical inferential systems.

The correlation between the scroll and two systems of reasoning has some important consequences. First, it explains why associative connections that emerge from the operation of S1 do look like inferences. Both systems are logical and presuppose the presence of guiding principles that model that reasoning. Second, the correlation specifies what is at issue in some classes of cognitive biases. Third, as the positive implicational fragment has good computational properties is makes an attractive candidate for experiments and models of reasoning in artificial general intelligence.

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