The intensional structure of epistemic convictions

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Abstract. We discuss an axiomatic setup as an appropriate account to the intensional structure of epistemic convictions. This includes a resolution of the problem of logical omniscience as well as the individual rendering of knowledge by different persons.

In this position paper we present a model for epistemic convictions, which provides the general framework for different applications as belief revision [6, 12], modalities [7–9], Frege's *mode of presentation* [10], and counterfactuals [11].

The idea is to model *knowledge* or *belief* of a person in an *axiomatic setup*. The purpose is to make good use of the concept of *derivation*, in fact *performed derivations*, to overcome some of the well-known problems of knowledge representation in formal frameworks.[1]

To avoid an intricate discussion of the conflicting terms of "knowledge" and "belief" we prefer to use the neutral designation of *epistemic conviction* for a person's knowledge or belief. In the last section, however, we address how our account may help to explicate, at least in part, the traditional understanding of *knowledge as true and justified belief*.

1 Axiomatic setup

We presuppose that epistemic convictions of a person can be represented by sentences in a formal(izable) language. We do not specify a particular formal language, but assume that, at least, propositional connectives and quantifiers are available.

Some basic convictions are fixed (for an individual person; they might differ from one person to another). A sentence belonging to these basic convictions

[1] Our approach reassembles ideas which one also finds in Doyle's *Truth Maintenance Systems* (TMSs), [4]. As TMSs were developed in the context of expert systems in Computer Science, they soon fell victim to complexity issues. For us it is, however, just the *qualitative* setup which matters from a philosophical point of view. The *quantitative* aspect may go out of control when one tries to explain and store every single step of a derivation.

is considered as an *axiom*, an axiom which can then be used to derive further epistemic convictions. A sentence is added to the person's convictions, if it is actually *derived* from the basic convictions by a *correct* derivation.

Remark 1. Here, the word "axiom" is not used in its traditional meaning as *evident truth*, but rather in the modern understanding as undisputed (or: indisputable) starting point for derivations.

Remark 2. It is an interesting question how such an axiom enters in the individual convictions of a person. Clearly, *empirical observation* will be one way; another way is by *learning in school*. However, a detailed discussion of this question is outside of the scope of this paper.

Remark 3. Our account also allows to study the effects of *incorrect reasoning* as well as the question, to which extent the rules for correct reasoning have to be part of the basic convictions. In the present paper, we presuppose that the rules of *correct* reasoning are an inherent part of a person's convictions. "Correct" should not be restricted to purely logical reasoning, but may include *inductive* reasoning in the way it is vindicated by common sense.

The axioms of a person's convictions are subject to changes. In the first place, one is continuously making new empirical observation and learning new facts. But one may also retract old convictions or revise the present ones. The dynamics of the change of convictions is a wide field, and here we aim only to argue that our axiomatic setup provides a tool to deal with it.[2]

2 Avoiding logical omniscience

Many approaches to knowledge and belief presuppose that it is closed under logical consequences. Although it comes sometimes under the label of *rationality criteria*, as in AGM [2], this presupposition is quite inadequate.[3] Alan Turing formulated this neatly [16, p. 451][4]:

> The view that machines cannot give rise to surprises is due, I believe, to a fallacy to which philosophers and mathematicians are particularly subject. This is the assumption that as soon as a fact is presented to a mind all consequences of that fact spring into the mind simultaneously with it. It is a very useful assumption under many circumstances, but one too easily forgets that it is false.

Under the heading of *logical omniscience*, this problem is acknowledged in the literature, but not much is done to solve it.[5] Even turning to a theory

[2] For an example how such belief revision might be implemented in our context, see [6].
[3] See [12] for a detailed discussion of AGM in our perspective.
[4] Also cited in [1, p. 261].
[5] The paragraph on logical omniscience in the article on Epistemic Logic in the Stanford Encyclopedia of Philosophy exposes here a certain helplessness [15, § 4].

of *implicit knowledge*, which is supposed to come from the logical closure of initial knowledge, is of little help. Just consider mathematical knowledge: it doesn't seem to be of particular interest to study a person's implicit knowledge of mathematics, which, in some sense, should contain the solutions of all open questions in Mathematics.[6]

In fact, Mathematics can serve as a guideline for the modelling of knowledge: we consider a theorem only as a part of a person's knowledge, when (s)he has a proof of it "at hand" or—as for every other knowledge—has learned it from a trustworthy source, in this case, from Mathematicians which did perform the proof of the theorem in question.

For belief, however, it is possible to be convinced of the truth of an open conjecture without having a proof yet. But such conjectures are explicitly *flagged* as such, and may enter the convictions of a person only as an "axiom" with lower credibility. At any account, *believing* in a conjecture might not be irrational, even if the conjecture is false. And if it is treated as an axiom of the person's convictions, there is no fundamental problem as long as no contradiction is actually derived.

In fact, it is probably only the derivation of a contradiction which is the trigger of what is called a *belief revision*.[7]

3 Individual structuring of convictions

The axiomatic setup exhibits the intensional structure of a person's convictions. These are not just the derived sentences, but the concrete derivations come into play in, at least, two ways: first, a derivable sentence has many different proofs in an axiomatic framework. Choosing one or another derivation can influence the trust in the derived sentences as well as the way the conviction can be defended when it comes under scrutiny. Secondly, and more important, the same set of sentences can be derived from different sets of axioms. Thus, extensionally equal sets of convictions can be represented by different sets of axioms.

We would like to illustrate the latter situation by a simplified example from Astronomy.

Person A may observe a good number of positions of the planets of our solar system, with the precision available in the 18th century. The coordinates of these positions are A's axioms, justified by empirical evidence. By intelligent inductive reasoning, A derives from these positions Kepler's laws for the movement of planets.

Person B learned at school Kepler's laws and, without ever looking to the sky at night, may derive the positions of the planets using some given initial data.

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[6] Barwise and Perry's *situation semantics* [3] is sometimes consider as a tool which might tame logical omniscience; while we consider it advantageous compared with possible world semantics, it is not clear how it should cope with examples from Mathematics.

[7] This aspect of our axiomatic setup is worked out in more detail in [6].

By construction, the astronomical knowledge of A and B should be equivalent. However, the difference in the intensional structure should become visible, when both learn about the more exact perihelion precession of Mercury as available in the 19th century. For A these are "just" new empirical data, which question, of course, the derived Kepler's laws, but which do not contradict the original axioms. For B, however, the very axioms are *falsified* and B's knowledge as such is called into question.

In [10] we mentioned as another example two axiomatizations of the natural numbers, the first one as a commutative semigroup, the second by the Peano Axioms. While in the former one, commutativity of addition is "built in", in the latter one this property requires a proof by induction. Thus, the *sense*—in Fregean terms—of a sum terms $t+s$ and $s+t$ is equal in the former but different in the latter axiomatic presentation.

In general, whenever one has two different axiomatizations which result in the same set of derived formulas, the very difference of the axiomatization can be considered as an intensional difference.

Due to the axiomatic nature of mathematical theories, they will provide many more examples to illustrate our point; you may think, for instance, of Geometry given in terms of points, straight lines, etc., following Euclid and Hilbert, or in terms of reflections, following Bachmann. In Physics, for instance, the Heisenberg picture [13] can be contrasted by the Schrödinger picture [14]. We expect that it should be possible to find examples even from "every day" concepts which may be represented extensionally equivalently, but intensionally differently.

4 The question "Why?"

The axiomatic setup is also the adequate model to study the way a person answers the question "Why do you believe this?" In general, the given answer should reveal the argument the person used in the derivation of the sentence in question. Only if this sentence is an axiom, the reason for its assumption as an axiom should be revealed.[8]

In fact, we can turn the perspective around, and use why-questions to uncover the axiomatic structure of a person's epistemic convictions.

A by-product of this analysis is that it gives support for the classical characterization of knowledge as *true and justified belief.* While the justification comes from derivations, the mentioned analogy to mathematical proof shows that, of course, the sentences used in the derivation need to be hereditary knowledge, i.e., being themselves all true. Leaving aside the notorious question how truth should be established, it rules out, at least, flawed justifications.

Again, we can draw on an example of Mathematics. When, in the last decades of the 19th century, the proof attempts of Kempe and Tait were considered by the mathematical community as correct proofs of the four colour problem, of course, this community did not have *knowledge* of the four colour theorem. Even

[8] See also Remark 2.

if we know today, by the proofs of the second half of the 20th century, that the four colour theorem is, indeed, true, the alleged proofs of the 19th century could not serve as justification, for the simple reason that they were flawed.

The problem confusing a correct justification with a raw justifiability, not respecting the heritability of truth was discussed a lot in context of the notorious Gettier examples, and the flawed knowledge based on it could well be called *Gettier knowledge*. When we subscribe the *No False Lemmas* condition, it goes without saying that also the “axioms” presupposed for the knowledge need to be true; this is not the case for the example discussed in [5, § 4].

Also the answer to the question “Why do you believe this ‘axiom’?” has to be taken into account, as also the axioms need to be justified. When somebody answers, he believes in the equation $E = mc^2$, “because I learned it in school”, we would consider it justified; an answer of the sort “because I read in my horoscope” would not serve as justification.

At the end, the answers to *why-questions* give us the relevant information to judge a person’s convictions, and such answers are supposed to be given along an axiomatic setup.

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