A Modal Semantics of Perception

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Abstract. This paper introduces a novel logical language for perception, aimed at formalizing both veridical and erroneous perceptual experiences. Building upon Jaakko Hintikka's approaches to perceptual statements within a modal framework, the paper argues in favour of including illusions and hallucinations in the formalization of perceptual experience. We explore the application of this formal modelling in robotics, specifically in visual verification, to help differentiating between correct and erroneous interpretations of sensor data, with implications for safety in autonomous systems.

Keywords: Perception · Illusion · Hallucination · Modal Language · Visual Verification

1 Introduction

Perception is the process through which any subject enters into contact with the external world. According to *the Representational Theory of Mind* [16, 23], perceptions are symbolic mental states through which the perceiver acquires information about the world. These internal representations have semantic properties, including *aboutness*, because they represent something relating to the external state of affairs by means of an internal vehicle.

The association between representational content and its external-world referent does not always succeed; it happens many times to misinterpret external triggers, and sometimes it happens to perceive things that do not exist in the external world. These types of unsuccessful perception are respectively referred to as illusory and hallucinatory perceptions.

Illusions are a wrong classification of the environmental input. Having the illusion of a cat means wrongly categorizing something as a cat, while in the external world there is a different physical object (for example, a pile of leaves which, in certain conditions, looks like a cat). During an illusion, the creative role of the brain takes over, and it misinterprets the inputs it receives from the external world, effectively creating an alternative reality. On the other hand, hallucination consists of cases in which the object of perception is completely made up by the agent's mind. The brain continuously generates internal images and thoughts, but it normally distinguishes these experiences from the perception of the external world. When this distinction deteriorates, internal images can be mistaken for reality (for example, during psychotic states). In this state, the agent does not have access to the information of the world, and she takes the realm proposed by the brain for granted. Hallucinating a cat is erroneously placing a fictitious object in the world.

Introspectively speaking, it is impossible for a perceiver to distinguish between successful and unsuccessful kinds of perception. This is clearly explained in the claim below:

Claim (Common Kind Claim, [4]). Veridical, illusory, and hallucinatory experiences (as) of an F are fundamentally the same; they form a common kind.

This entails that, for any veridical perception of an ordinary object, we may imagine a corresponding illusion or hallucination indistinguishable by introspection [4]. Without a way to distinguish truth from error, an agent's perceptions could be wrong at any time. This brings us to the conclusion that, at least in principle, the agent always has access to an alternative reality made up by the agent's brain where the perceived object is misinterpreted.

The literature in formal logic has attempted so far to treat veridical perception as a primary activity, dismissing illusions and hallucinations. But the Common Kind Claims suggests that there is an essential resemblance among illusions, hallucinations and veridical perception which is not contingent. Although illusions and hallucinations are erroneous perceptual experiences, the perceptual activity continues to occur. Erroneous cases of perception are structural and defining in the perceptual experience to such an extent that they must be reconsidered in any attempt to formalize such activity.

Introducing illusions and hallucinations as kind of perception serves not only from a theoretical perspective, but also in the real-case scenario of visual verification. Just like humans, visual inspectors cannot differentiate between accurate and faulty perceptions. This has significant implications for safety in critical scenarios where reliance on autonomous agents for visual recognition strongly requires verification of their outputs to prevent the risk of serious accidents resulting from errors during perceptive data classification.

In this paper we provide a new formal language for perceptual statements which aims at defining veridical, illusory and hallucinatory perceptions, breaking with a long-standing tradition in formal methods which does not recognize incorrect perception. The paper proceeds as follows. Section 2 shows the erroneous prerequisite found in the literature in the formalization of perception. Section 3 investigates visual verification and the issues it faces. Section 4 introduces the new formalization, which includes unsuccessful cases of perception. We conclude with some remarks for future research.

2 Related Work

In the context of a general theory of propositional attitudes, Hintikka proposes the formalization of perceptual statements [9]. He submitted the logic of perception as a branch of modal logic. Letting a be an agent and q a proposition, the statement that refers to the perceptual relation between a and q is:

Proposition 1. $\mathcal{P}_a q =$ *a perceives that* q

where $\mathcal{P}$ is a modal operator. The truth condition for a perceptual statement is formulated as follows:

Definition 1 (Truth Value Condition 1). *The sentence in Proposition 1 is true in world w if and only if q is true in all possible worlds w' which are compatible with what a perceives in world w [15, p. 37].*

The compatibility between possible worlds is the condition that establishes an *alternative relation* in the domain of possible worlds. World w is compatible with w' for Agent a when w' is among the worlds that a considers conceivable when she is in w. That makes the alternative worlds accessible one another for agents. Hence, the statement "a perceives that q" is true in w if and only if q is true in all accessible perceptual alternatives w' of the actual world w [15, p. 38].

In a semantics in which the meaning of statements emerges by analysing more than one possible world, problems arise because a single term in a perceptual statement could refer to different objects in such multiple states of affairs [9]. In Hintikka's words, modal contexts exhibit *referential multiplicity*. By Definition 1, q must be a member of all (possible) states of affairs alternative to the actual world, and it must be recognized by Agent a in all such alternative worlds. This becomes a noticeable problem when the *existential generalization* is introduced.

Existential generalization consists of replacing any statement with a free singular term of the type $F(a)$ in a statement with a variable bounded to an existential quantifier like $\exists(x)F(x)$. Exclusively referring to the actual world, such replacement is licit because when a singular term a satisfies the predicate $F(.)$, it entails that *something* with that predicate exists. Thus, sticking to the actual world, if $F(a)$ is true, then $\exists x F(x)$ is true because there exists a term which makes true the formula $\exists x F(x)$, i.e., a itself. This mode of inference breaks down in the modal context since the perceptual proposition $\mathcal{P}_a q$ cannot be existentially generalized as $\exists x \mathcal{P}_a x$ because it could be the case that the term q does not refer in each alternative possible world to the same individual [9]. If individuals are members of more than one state of affairs, they could have as a reference in each of these states of affairs a different object, which leads to the untenability of the existential generalization [9].

According to Hintikka, to make the existential generalization possible, "we have to assume that the term with respect to which we are generalizing [...] refers to the same individual in all the different "worlds" we are considering" [9, p. 160]. This requirement, which we will call Hintikka's strong condition solution, needs an explicit formal form:

Definition 2 (Strong Condition Solution, [9]). $(\exists x)(\mathcal{P}_a(q{=}x)$ *and* $(q{=}x))$.

in which the first conjunct guarantees that the term "q" refers to the same entity in all possible worlds compatible with what a perceives, and the second conjunct extends the uniqueness of the reference of "q" to the actual world [9]. Hintikka stresses that, in the particular case of perceptual statements, it is possible to use another formalization:[3]

Definition 3 (Strong Condition Solution I, [9]). $(\exists x)(\mathcal{P}_a(q = x))$

Definition 3 is equivalent to 2 "when the veracity of [a's] perceptions is not at issue" [9, p. 160], namely it is already settled that "$q = x$" holds in the actual word w, i.e., $w \vDash (q = x)$. According to Hintikka, "it is required that one can perceive only what is in fact the case" [9, p. 160], meaning that perception can only be non-erroneous and non-illusory. The formalization perfectly reflects his idea of what perception is: a structure of consciousness "always mediated by conceptual schemes" [15, p. 39] and which "involves causal interaction with external objects" [15, p. 39].

Not doubting the truth of q in the actual world is nothing other than the condition of *reflexivity* on the alternative relation in the domain of possible worlds. Assuming "$q = x$" in the actual world primarily means that the agent has access to the information of the actual world from the actual world itself.

Notice that Strong Condition Solution is what allows any perceptual statements to be true. Given Definition 1, without the certainty that in any alternative world w' the term q refers to the same individual, perceptual statements would be unassessable, or even false. Evaluating Proposition 1 in w, if in one alternative world w' in the model q refers to a different individual, then $\mathcal{P}_a q$ is false because q should be truthfully recognized in every w' compatible with w, like posed by Strong Condition Solution and by Definition 1. Relaxing the Strong Condition Solution is one of the aims of this paper, which would lead to a change in Hintikka's truth value condition for perceptual propositions to widen the validity of perceptual statements.

The Hintikka conditions that pose difficulties for a formalization encompassing erroneous cases of perception are imposed veridicality and reflexivity. In Hintikka's account, perception is valid only if it is truthful. A perceptual proposition to be true requires the presence of the same object in every relevant possible world in the logical model, including the actual one, making non-perception such cases where the same object is not present in one of these worlds. In this framework, illusions and hallucinations are not regarded as perception at all, because they are cases of erroneous perception in which the object changes in the alternative worlds of the model.

During illusions, the mistake the agent does in the actual world perceiving something wrong amounts to having access to "another" world made up by the agent's brain where the actually perceived object of perception is different. Despite the error, and despite the fact that in the two alternative worlds the objects are different, it cannot be said that the agent is incapable of perceiving. Perception remains valid because the process of representing the surrounding is still in

[3] The statements in Definition 2 and 3 are *identification statements* since the agent a can be said to have perceptually identified q because she attributes to q the same individual x in each possible world related to the actual world.

progress and, at this level, the agent is unable to distinguish it from a truthful perception (see *Common Kind Claim*).

Moreover, perception is valid only if it has access to the actual world. Another consequence of the truth value condition proposed by Hintikka is the necessity of referencing the actual world in the analysis of the model. The *reflexivity* condition on the accessibility relation posed by the Strong Condition Solution forces the perceiver to have access to every possible world of the model from itself, including the actual world.

However, as can be seen in the case of hallucinations, the agent is not always able to access the information of the actual world, yet this does not diminish the fact that an act of perception is occurring. During a hallucination, the agent completely makes up the object of perception without referring to external information. Regardless, the act of perceiving is not denied just because the semantic content does not have a currently existing referent, but one invented by the perceiver. Once again, introspectively speaking, the experience of perceiving is kept and indistinguishable from truthful one; therefore, hallucinations must be considered in the formalization of perception.

Classifying erroneous perception cases as non-perception is an a priori stance that not only needs to be argued, but should be avoided. In fact, the agent's ability to perceive objects does not diminish when what is perceived is incorrect or misinterpreted. Human perception is a complex activity that not only depends on the agent's access to information of the external world, but it is also guided by a potentially fallacious categorization activity within the agent. Failing to acknowledge that agents may incur errors during perception amounts to analysing perceptual activity only partially. Therefore, illusions and hallucinations are modes of perception that must be considered if one seeks to fully formalize perceptual activity.

For that reason, we are opening up the possibility that in alternative worlds, there may be something different from the truthfully perceived object. Taking inspiration from Robert Howell's analysis of statements expressing visual recognition [10], we steer the discussion in a direction where, for perception to occur, it is not necessary for the same object to exist in all possible worlds of the model. Howell argues that it is not a problem for a perceptual statement to be true that the observed object is recognized differently among the plurality of perceptual alternatives. In this way, Howell starts relaxing the strong requirement imposed by Hintikka, opening up the possibility that in alternative worlds, there may be something slightly different from the truthfully perceived object.

Howell's position should be strengthened further to the point of concluding that, for a perceptual statement to be true, it is sufficient for the presence of one relevant object in one possible world accessible by Agent a. What illusions and hallucinations show is that not only having access to alternative worlds in which there are different objects is a prerequisite for perception, but also that the presence of something different in these alternative worlds is what makes perception possible. During illusion, the perceiver has access to the actual world, in which there is the well-interpreted object, and also to an alternative one, in which there is the illusory object. Something similar happens during hallucinations: the agent perceives the hallucinatory object in an alternative reality to which she has access. Hallucinations and illusions demonstrate that it is no longer necessary for the exact same perceived object to be present in every world of the model; it is enough for the perceived object to exist in only one world of the model in order for perception to occur. Formally:

Definition 4 (Truth Value Condition 2). *Proposition 1 is true in world w if and only if q is true in at least one possible world w' (including w itself) compatible with what a perceives in world w.*

The consequences of this change are significant. The new truth value condition for perceptual statements would include not only veridical perception, as Hintikka and Howell posed, but also erroneous cases of perception such as illusions and hallucinations. In this way, perceptual ability is exhaustively described in all its forms, not just in the truthful one.

3 An Application Scenario

By discussing a new truth value condition for perceptual statements and the consequent new formal language for perceptual statements, this paper provides insights that can be implemented in visual verification. Visual verification is the process of determining the presence or the absence of an object in the data collected from the environment through visual inspection [11]. It can be performed manually by a human observer, but, in this context, we are interested in its automation in object detectors.

An object detector is an artificial intelligence (AI) system that identifies and classifies objects within an image or video. Object detectors are implemented in a vast number of robots. Such robots integrate AI-driven vision systems that enable them to perceive and interact with their environment. They are commonly used in industrial robots, for detecting and handling objects in manufacturing and assembly lines; in autonomous vehicles, for identifying pedestrians, traffic signs, and obstacles [2, 6], in spacecraft robotic vehicles, for increasing their autonomy in space mission [13, 12], but also in service robots, “doing the most mundane household activities” [20, p. 197].

For doing so, robots are equipped with sensors like video cameras [20], LiDARs (Light Detection and Ranging) [8], thermal cameras, radars, and ultrasonic sensors [6] to detect and recognise objects. Once sensors gather outside information, the control panel extracts the image to be sent to Computer Vision (Figure 1). Computer Vision operates using YOLO algorithm, developed by J. Redmon, S. Divvala, R. Girshick, and A. Farhadi in 2016 [18]. YOLO (You Only Look Once) is a one-stage CNN-based object detection algorithm [6] through which the input image coming from the sensors is divided into an $n \times n$ grid. Firstly, “[t]he cell of the grid containing the center of an object in input image is responsible for its detection” [20, p. 202]. Secondly, the algorithm “returns

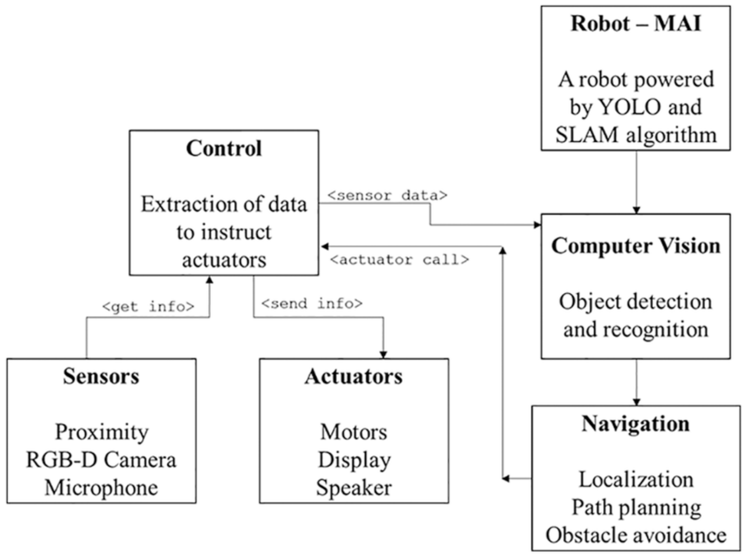

Fig. 1. Navigation framework of MAI, a robot powered by the YOLO and SLAM algorithms [20, p. 202].

inference in terms of bounding boxes of different colors with labels for different objects as shown in Fig.[2]" [20, p. 206], that detects all instances of objects from several classes [1]. In Figure 2, yellow boxes classify people, red ones identify chairs, and so on.

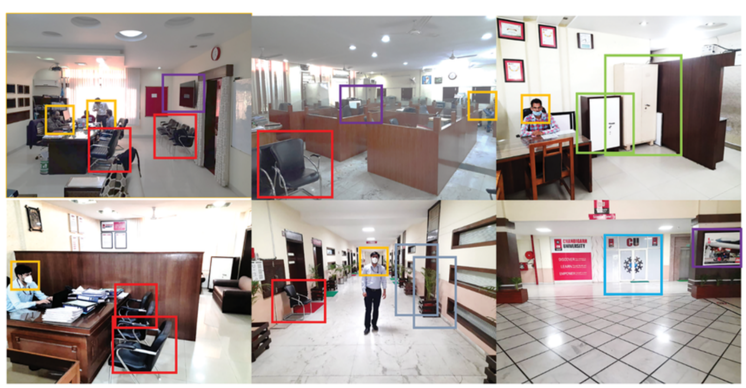

Fig. 2. Results for object detection using YOLO in different rooms and corridors of Chandigarh University [20, p. 205].

Despite the advanced technology, object detection faces challenges in recognizing objects. For example, the object could be viewed from different angles, it could be of different shapes or sizes, and it could adopt different speeds. The object could be obscured by other objects, and it is subject to lighting condition alterations (too low, or too bright). Sometimes, there are many objects in an image, making the object to be perceived less visible to the detector. These challenges could lead to a wrong interpretation of the surroundings [21].

In particular, LiDARs are sensitive to attackers, i.e., entities that exploit LiDAR systems to deceive, disrupt, or compromise their data. While analysing the surroundings, the LiDAR sensor transmits a laser pulse that reflects back to the LiDAR when it hits an object [19]. If, during this process, the LiDAR is deceived by an attacker, the return signal is altered to produce errors in the representation of the world [19]. That can be compared to a human illusion.

Moreover, "deep object detectors can hallucinate non-existent objects, and they may even detect those missing objects at their expected location in the image, see Fig. [3]" [11, p. 2234]. Even though the ability to hallucinate non-existent objects may seem useful for visual inspection applications, "the costs for hallucinating missing objects are higher than missing existent objects" [11, p. 2234]. Indeed, while the cost for not detecting an existing object (false negative) is that humans must inspect the detection, hallucinating a non-existing object (false positive) may cause catastrophes [11]. We leave to the reader's imagination the consequences for a driverless vehicle of hallucinating a pedestrian crossing the road on a high-speed highway.

The challenges linked to such redundant detections [22] that lead to the erroneous perception of "ghost" objects [8] in object detection are faced by automatic visual verification [11]. In [5] the problem of formal verification for object detection attacks is characterised in three types:

1. Misdetection: the attack can occur when there exists a ground truth bounding box which is not detected at all;
2. Misclassification: the attack can occur when there exists a ground truth object which is not classified correctly;
3. Overdetection: the attack can occur when a concrete input with a bounding box which does not exist in the ground truth is found.

Assistance for visual verification could come from the new formalization proposed in this paper. In particular, the implementation of the formalization of the different types of perception proposed in Section 4 in automatic visual verification could help to discern between veridical detection, illusory detection for misclassification and hallucinatory detection for overdetection.

4 A First Order Language for Perception

This Section introduces the first-order logic $\mathcal{L}^P$ for perception, with its syntax and its semantics. This new language is more expressive than Hintikka's as it introduces different types of perception operators according to the analysis shown in Section 1.

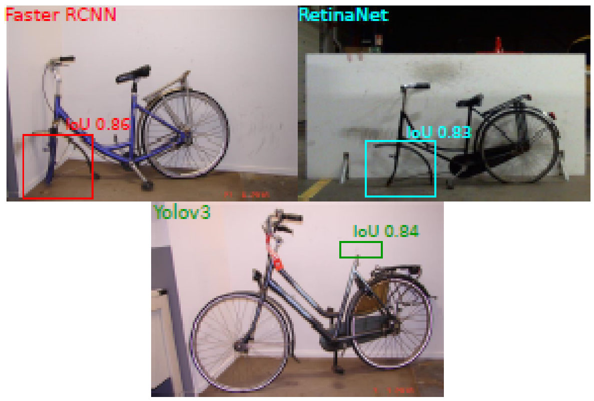

Fig. 3. Hallucination examples on DelftBikes for Faster RCNN, RetinaNet, and YOLOv3. Faster RCNN and RetinaNet detect the front wheel and YOLOv3 predicts the saddle with a high IoU score. Deep object detectors may detect non-existent objects at their expected location [11, p. 2234]

Definition 5 (Syntax).

$$B := \{p, q, ..., r\}$$

$$V := \{x, y, ..., z\}$$

$$A := \{P_1(t_1, ..., t_n), ..., P_n(t_1, ..., t_n)\}$$

$$E := \{\neg, \wedge, \vee, \rightarrow\}$$

$$Q := \{\forall, \exists\}$$

$$M := \{\mathcal{P}_{a\in\mathcal{A}}, v\mathcal{P}_{a\in\mathcal{A}}, i\mathcal{P}_{a\in\mathcal{A}}, h\mathcal{P}_{a\in\mathcal{A}}, \}$$

$$\mathcal{A} := \{a, b, ..., c\}$$

These respectively represent the set B of objects; a set V of variables for such objects in perceptual propositions; a set A of atomic predicative sentences constructed by non-logical predicates $P_i(.)$ and terms t_i; the set E of logical connectives; the set Q of quantifiers; the set M that contains the *perceptual modalities*, respectively $\mathcal{P}_{a\in\mathcal{A}}$ the generic perception modality, $v\mathcal{P}_{a\in\mathcal{A}}$ for veridical, $i\mathcal{P}_{a\in\mathcal{A}}$ for illusory and $h\mathcal{P}a \in \mathcal{A}$ for hallucinatory perception, all indexed by elements in the set $\mathcal{A}$ of agents.

Definition 6 (Language). *Elements of the perceptual language $\mathcal{L}^P$ are defined as:*

$$\phi := P(t_1, ..., t_n)$$
$$\psi := \mathcal{P}_a(\phi) \mid v\mathcal{P}_a(\phi) \mid i\mathcal{P}_a(\phi) \mid h\mathcal{P}_a(\phi)$$
$$\xi := \phi \mid \psi \mid \neg\xi \mid \xi \wedge \xi \mid \xi \vee \xi \mid \xi \rightarrow \xi$$
$$\theta := \forall x \xi(x) \mid \exists x \xi(x)$$

The language $\mathcal{L}^P$ includes a set of formulas ϕ including predicative formulas and quantified formulas. The metavariable ψ extends it to define the new formalization for perceptual statements, including the modal operators for a generic perceptual statement and more specific operators for veridical, illusory and hallucinatory perception for an agent, ranging over formulas in ϕ. The perceptual modal operators range over possibly complex predicates of objects; quantified formulas have in their range both predicative and perceptual formulas. Notice that we explicitly exclude iteration of perception modalities. All formulas are closed under negation, conjunction, disjunction and implication. We further include quantified statements where the notation $\xi(x)$ in the scope of the quantifier denotes a predicative, respectively perceptual statement, or eventually their closure under logical operators where the variable x is bounded.

Example 1. Some examples:

- Agent a perceives a man as Mr Smith: $\mathcal{P}_a(Man(t) \wedge Smith(t))$
- There is an object such that Agent a perceives it as a man who is Mr. Smith: $\exists x(\mathcal{P}_a(Man(x) \wedge Smith(x))$
- Agent a perceives all objects as flying pigs: $\forall x(\mathcal{P}_a(Pig(x) \wedge Flying(x))$
- There is an object such that Agent a perceives it as a man who is Mr. Smith, and that object is a man who is Mr. John: $\exists x(\mathcal{P}_a(Man(x) \wedge Smith(x))) \wedge (Man(x) \wedge John(x))$

To extend further these examples, we want to state that Agent's a perception is veridical, illusionary or hallucinatory and provide distinct semantic evaluation clauses. We start with a definiton of the modelling structure.

Definition 7. *A model $\mathcal{M}$ is defined as a tuple*

$$\mathcal{M} = \{W, R_a, L(.)\} \tag{1}$$

where:

- *W is the finite set of all possible worlds;*
- *$R_a \subseteq W \times W$, the accessibility relation among worlds labelled by an agent $a \in \mathcal{A}$, such that its properties – and in particular whether wRw for all $w \in W$ – are specified in function of the perceptual operators;*
- *$L : W \rightarrow \mathcal{L}^P$ is a labeling function, which associates at each world the formulas that are true in such a world.*

Definition 8 (Semantic clauses for non-modal formulas). *Given a model $\mathcal{M}$ and a formula ϕ, ϕ or its predicative counterpart θ is true at a world w according to the following inductive definition:*

$$\mathcal{M}, w \vDash \top$$
$$\mathcal{M}, w \nvDash \bot$$
$$\mathcal{M}, w \vDash P(t_1, ..., t_n) \textit{ iff } P^{\mathcal{M}}(t_1^{\mathcal{M}}, ..., t_n^{\mathcal{M}}) \in L(w)$$
$$\mathcal{M}, w \vDash \neg\phi \textit{ iff } w \nvDash \phi$$
$$\mathcal{M}, w \vDash \phi_1 \wedge \phi_2 \textit{ iff } w \vDash \phi_1 \textit{ and } w \vDash \phi_2$$
$$\mathcal{M}, w \vDash \phi_1 \vee \phi_2 \textit{ iff } w \vDash \phi_1 \textit{ or } w \vDash \phi_2$$
$$\mathcal{M}, w \vDash \phi_1 \rightarrow \phi_2 \textit{ iff } w \vDash \phi_2 \textit{ whenever } w \vDash \phi_1$$
$$\mathcal{M}, w \vDash \forall x P(x) \textit{ iff for all } t \in B, w \vDash P(t)$$
$$\mathcal{M}, w \vDash \exists x P(x) \textit{ iff it exists } t \in B \textit{ such that } w \vDash P(t)$$

We now present the conditions for the truth value of any perceptual statement. Perceptual statements may include not only veridical perception but also illusions and hallucinations.

Definition 9 (Semantic clauses for perceptual formulas). *Given a model $\mathcal{M}$ and a formula ψ, ψ or its predicative counterpart θ is true at a world w according to the following inductive definition:*

$$\mathcal{M}, w \vDash \mathcal{P}_a(\phi) \textit{ iff } \exists w' \textit{ such that } R_a(w, w') \textit{ and } w' \vDash \phi$$
$$\mathcal{M}, w \vDash v\mathcal{P}_a(\phi) \textit{ iff } \mathcal{M}, w \vDash \mathcal{P}_a(\phi) \textit{ and } w' = w$$
$$\mathcal{M}, w \vDash i\mathcal{P}_a(\phi) \textit{ iff } \mathcal{M}, w \vDash \mathcal{P}_a(\phi) \textit{ and } w' \neq w \textit{ and } w \vDash \neg\phi$$
$$\mathcal{M}, w \vDash h\mathcal{P}_a(\phi) \textit{ iff } \mathcal{M}, w \vDash i\mathcal{P}_a(\phi) \textit{ and } \neg R_a(w', w)$$
$$\mathcal{M}, w \vDash \neg\psi \textit{ iff } w \nvDash \psi$$
$$\mathcal{M}, w \vDash \psi_1 \wedge \psi_2 \textit{ iff } w \vDash \psi_1 \textit{ and } w \vDash \psi_2$$
$$\mathcal{M}, w \vDash \psi_1 \vee \psi_2 \textit{ iff } w \vDash \psi_1 \textit{ or } w \vDash \psi_2$$
$$\mathcal{M}, w \vDash \psi_1 \rightarrow \psi_2 \textit{ iff } w \vDash \psi_2 \textit{ whenever } w \vDash \psi_1$$
$$\mathcal{M}, w \vDash \forall x \psi(x) \textit{ iff for all } t \in B, w \vDash \psi(t)$$
$$\mathcal{M}, w \vDash \exists x \psi(x) \textit{ iff it exists } t \in B \textit{ such that } w \vDash \psi(t)$$

The generic perception operator $\mathcal{P}_a$ weakens Hinitikka's requirement of compatibility in *all* possible worlds accessible from the actual one according to agent a, to the existence of at least such one world compatible with the actual one where the object of perception ϕ is true. This is then declined in the various types of perception, erroneous or not, to provide an exhaustive description of the perceptual activity. The variants entail the inclusion of additional conditions to detail the case at hand.

Veridical perception according to the operator $v\mathcal{P}_a$ adds the condition that the world in which the object of perception holds is the actual one. This in turn entails the existence of a reflexive relation R_a. Notably, this simply recasts

Hintikka's truth condition of the presence of ϕ in every world in the model when no alternatives are taken into account. Note that this definition has as major consequence that all true sentences in the world of perception become object of veridical perception.

The further condition of the operator $i\mathcal{P}_a$ for illusory perception requires that the object of perception ϕ does not hold in the actual world in w, while it still holds in an alternative world. Unlike cases of truthful perception, the agent a illusorily perceives something that belongs to an alternative world, although she has access to the actual world. Hence, an object t such that $P(t)$ holds in the actual world is perceived as $Q(t)$ – and $\neg P(t)$ in an alternative world which is accessed from the actual one.

The further condition for hallucinatory perception by the operator $h\mathcal{P}_a$ posits along the absence of ϕ in w, that the accessibility relation R_a is irreflexive and anti-symmetric. An illusion with the addition of lack of reflexivity and symmetry makes the agent a incapable of gathering information about the actual world, as it happens in hallucinations, while perceiving something in an alternative world. Hence, an object t such that $P(t)$ holds is perceived in an alternative world which has no access to the actual one.

Evaluation for perceptual statements is closed under logical connectives, so as to be able to evaluate formulas that mix different types of perceptions. The closure applies also to ξ formulas, not explicitly given here, to allow to build combinations of perceptual and non-perceptual statements. Quantified perceptual formulas are abbreviations for statements of the form: $\forall x\mathcal{P}_a(Q(x))$, respectively $\exists x\mathcal{P}_a(Q(x))$, i.e. "Agent a perceives everything as Q", respectively "There is something that Agent a perceives as Q".

Example 2. In this example, we propose a model $\mathcal{M}$ that formalizes the process of detection and recognition, as shown in Figure 3, and visual verification. The model $\mathcal{M}$ is composed by:

1. the set of possible worlds $W = \{w_1, w_2, w_3\}$; we assume w_1 to be the designated actual world;
2. the object detectors, $\mathcal{A} = \{a, b, c\}$;
3. a set of formulas predicating certain properties $A = \{H, S, W\}$ of objects $B = \{p, q, r\}$: an handlebar $H(q)$, a saddle $S(p)$ and a bicycle wheel $W(r)$;
4. the accessibility relations for respectively the agent a, b and c, which state what each object detector can see:
   – $R_a = \{< w_1, w_1 >\}$;
   – $R_b = \{< w_1, w_2 >, < w_2, w_1 >\}$;
   – $R_c = \{< w_1, w_3 >\}$;
5. the following labelling function respectively for w_1, w_2 and w_3, i.e. which objects are present in which state:
   – $L : w_1 \rightarrow \Gamma$ with $\Gamma = \{H(q)\}$;
   – $L : w_2 \rightarrow \Sigma$ with $\Sigma = \{W(r)\}$;
   – $L : w_3 \rightarrow \Delta$ with $\Delta = \{S(p)\}$.

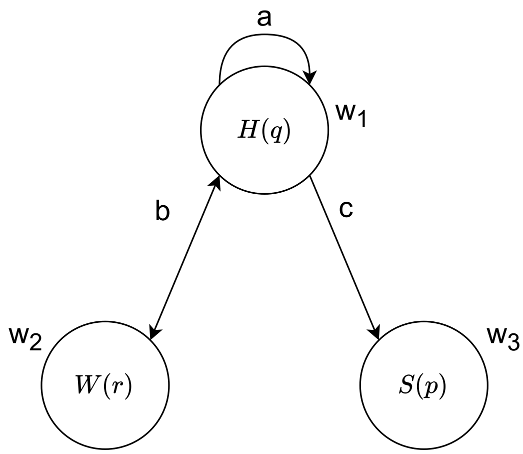

Fig. 4. Model $\mathcal{M}$

The process of detection and recognition made by object detectors consists in realizing the conditions under which the formula $\mathcal{P}_y(\exists x Q(x))$ can be made true, for some value of the $y \in \mathcal{A}$, of predicate $Q \in A$ and variable $x \in V$ to be substituted for an element $t \in B$.

In model $\mathcal{M}$ of Figure 4, as $R_a(w_1, w_1)$ and $w_1 \vDash H(q)$, detector a has veridical perception that there is a handlebar, hence $w_1 \vDash v\mathcal{P}_a(\exists x H(x))$. Moreover, $w_1 \vDash i\mathcal{P}_b(\exists x W(x))$, i.e. detector b has illusory perception that there is a wheel, since the detector has access to an alternative world $R_b(w_1, w_2)$ such that $w_2 \vDash W(r)$ but $w_1 \nvDash W(r)$. Finally, $w_1 \vDash h\mathcal{P}_c(\exists x S(x))$, i.e. detector c has hallucinatory perception that there is a saddle, because: $R_c(w_1, w_3)$ but $\neg R_c(w_1, w_1)$, $\neg R_c(w_3, w_3)$ and $\neg R_c(w_3, w_1)$; also, $w_1 \nvDash S(p)$ but $w_3 \vDash S(p)$.

5 Axiomatization

As for many epistemic notions formalized in a modal setting, the question arises as to whether perception sits in a novel conceptual space, or how it interacts with more standard notions of knowledge and belief. This has been, for example, the case of information [7, 17, 3].

Hintikka's Definition 1 does not offer a sufficiently fine-grained distinction from knowledge in normal modal logic $S4$. This suggests that "a perceives that q" must be true if and only if "a knows that q" is true, excluding indeed all cases of non-veridical perception. Moreover, it must be possible that "a believes that q" is true while "a perceives that q" is false, creating the conditions for beliefs based on other means than perception. These brief observations suggest that while Hintikka's notion of perception can be disentangled from mere belief, it allows only knowledge from veridical perception, and the latter always implies the former. This excludes the interesting cases of belief based on non-veridical perception we have considered. On the other hand, Definition 4 has the opposite problem of collapsing perception with belief, thereby making the former insufficient for knowledge even when veridical. We offer here some indications on the potential axiomatization for our semantics of perception independent from that of knowledge and belief.

5.1 Axiomatization of Simple Perception

Our $\mathcal{P}_a$ operator for simple, unqualified perception is a modality for which axiom K is satisfied. Moreover, generalizing the argument for irriflexivity in [14], it is possible to show that the frame condition that we will call *weak seriality* and expressed as $\exists w \exists w', wRw'$ is conservative over K. Hence, we need to impose an additional condition on the frame $F = \{W, R\}$, namely that $R \neq \emptyset$. Hence, our axiom is:

$$K^{\mathcal{P}} : \mathcal{P}_a(\phi \to \psi) \to (\mathcal{P}_a\phi \to \mathcal{P}_a\psi)$$

5.2 Axiomatization of Veridical Perception

The class of frames for veridical perception is closed under reflexivity $(\forall w, wRw)$, under standard seriality $(\forall w \exists w', wRw')$, and symmetry $(\forall w, w'(wRw' \to w'Rw))$. Reflexivity makes veridical perception factual. Seriality makes explicit that veridical perception is perception (as standard seriality implies weak seriality), a new axiom we simply call vP. Finally, symmetry underlines that veridical perception is coherent. Hence, the following schemas are valid:

$$T^{v\mathcal{P}} : v\mathcal{P}_a\phi \to \phi$$
$$vP : v\mathcal{P}_a\phi \to \mathcal{P}_a\phi$$
$$B^{v\mathcal{P}} : \phi \to v\mathcal{P}_a(\neg v\mathcal{P}_a\neg\phi)$$

$T^{v\mathcal{P}}$ reads "If a veridically perceives that ϕ, then ϕ is true" in the world of perception. $B^{v\mathcal{P}}$ reads "If ϕ is true, then a veridically perceives that she cannot veridically perceive its negation".

5.3 Axiomatization of Illusory Perception

For illusory perception, the class of frames is not reflexive $\neg(\forall w, wRw)$ as we do not want veridicality, weakly serial, and symmetric. Symmetry allows illusory perception to be revised by comparison with what is factual in the actual world.

$$T^{\perp} : i\mathcal{P}_a\phi \to \neg\phi$$
$$iP : i\mathcal{P}_a\phi \to \mathcal{P}_a\phi$$

$$B^{i\mathcal{P}} : \phi \rightarrow v\mathcal{P}_a(i\mathcal{P}_a\neg\phi)$$

Notice that $B^{i\mathcal{P}}$ is a better formulation of $B^{v\mathcal{P}}$, reads "If ϕ is true, then a veridically perceives that she illusory perceives its negation".

5.4 Axiomatization of Hallucinatory Perception

For hallucinatory perception, the class of frames is weakly serial, not reflexive and asymmetric $\exists w, w'$ *such that* $wRw' \wedge w'\neg Rw$ (note that being irreflexive it cannot be antisymmetric). This, in turn, means that the following schemas are valid:

$$T^{\perp} : h\mathcal{P}_a\phi \rightarrow \neg\phi$$

$$hP : h\mathcal{P}_a\phi \rightarrow \mathcal{P}_a\phi$$

$$B^{\perp} : \phi \rightarrow \neg v\mathcal{P}_a(i\mathcal{P}_a\neg\phi)$$

which together could be phrased as follows: "If a has hallucinatory perception that ϕ, ϕ is false in the world of perception, but it is true somewhere, and she has not veridical perception that she has an illusion that ϕ". This explains hallucinations as false perception which cannot be recognized as such in the real world.

To sum up: all qualified forms of perception (veridical, illusory, hallucinatory) imply truth of the content of perception *somewhere*; the distinction between veridical and non veridical perception is based on the actual world of perception and it is expressed as the failure of axiom T; additionally, veridical and illusory perceptions have a means of verifying what is perceived, while hallucinatory perception implies unawareness of the current perception (i.e. the impossibility to connect the perception to the real world).

In general, the relationship among the modal operator of perception and the standard ones for knowledge and belief can be understood as follows: on the one hand, perception is weaker than knowledge because P_a is not always factual, in particular in the illusory and hallucinatory forms, and it is not positively introspective (transitivity never holds for P_a); on the other hand, perception is stronger than belief, as it not only allows model where some world satisfies a formula while some others its negation as with standard belief, but also it expresses such distinction in a finer way allowing concurrently $vP_a(\neg\phi)$ and $iP_a(\phi)$. Moreover, unlike belief, perception allows a form of negative introspection ($B^{\perp}$) in hallucinations. Briefly, perception can be seen as a cognitive activity situated midway between knowledge and belief, a view that positively aligns with our intuitive understanding of what perception in its different forms is.

6 Conclusion

We discussed a modal interpretation of perceptual statements in which illusions and hallucinations are not marginal anomalies; they are structurally integral to perceptual experience. This perspective has valuable applications in visual verification. Object detection often faces challenges that undermine the accuracy and reliability of the objects' classification. Implementing this new formal approach could aid in distinguishing between veridical and erroneous perceptions, reducing the risk of serious errors arising from perceptual mistakes.

Future research will provide: first, alternative definitions that may reduce undesirable behaviours, such as veridically perceiving everything that is true, or add interesting behaviours such as the possibility of veridically and illusory perceiving the same content; second, appropriate meta-theoretical results; third, extensions of this model in establishing perception under uncertain conditions, by adding weights to the perception operators in order to allow for the user to prioritize among various perceptual options; fourth, formal verification of veridical, illusory or hallucinatory perception in applications, by providing a model checking procedure which associates illusory perception to misclassification and hallucinatory perception to overdetection.

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