A Depth-Bounded Semantics For Becoming Informed

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Goals

Our purpose is to develop a logic that satisfies the following constraints:

- Agents have limited resources;
- Agents can expand their information dynamically, i.e. by acquiring new contents through communication;
- Agents are able to distinguish between trustworthy and untrustworthy communications.

Background

Depth-bounded boolean logic (DBBL): D'Agostino (2015)

- Logic for single-agent reasoning
- An agent is characterized by a depth-bounded consequence relation: $\vDash^0, \vDash^1, \ldots, \vDash^k, \vDash^{k+1}$
- The parameter k represents the ability to carry out complex inferences

Multi-agent depth-bounded boolean logic (MA-DBBL): Cignarale and Primiero (2020)

- Extends DBBL for multi-agent reasoning via the two modal operators of "becoming informed" and of "being informed"
- It allows no secrecy

Model I

A model for DBBL-BI$_n$ is a tuple:

$$\mathcal{M} = ((\mathcal{S}^0, +), \mathcal{A}, \{R_i\}_{i \in \mathcal{A}}, \preceq, \{R_{i,j}\}_{\{i,j\} \subseteq \mathcal{A}}, \mathcal{P}, v)$$

- $\mathcal{S}^0 = \{s, s', s'', ..., s^n\}$ is a finite set of basic informational states;
- $+$ is a composition function s.t.:

$$\mathcal{S}^{n+1} = \mathcal{S}^n \cup \{s' + s'' \mid s', s'' \in \mathcal{S}^n\};$$
$$\mathcal{S} = \bigcup_{n \in \mathbb{N}} \mathcal{S}^n;$$

- $\mathcal{A} = \{i, j, ..., h\}$ is a finite set of agents;
- $\preceq$ is a preorder over $\mathcal{A}$;
- R_i and $R_{i,j}$ are accessibility relations;
- $\mathcal{P} = \{p, q, ..., r\}$ is a finite set of propositional variables;
- $v : \mathcal{S} \mapsto (\mathcal{P} \rightharpoonup \{1, 0\})$ is the valuation function.

Model II

- $R_i \subseteq \mathcal{S} \times \mathcal{S}$ is a preorder such that:

  if i can access a composite state $s + s'$ then i can access both parts;

  if i can access to s and s', then she can access to their composition $s + s'$;
- $R_{i,j} \subseteq \{(s', s) \mid (s', s') \in R_i, (s, s) \in R_j\}$;

- $(s, s') \in R_G$ iff $(s, s') \in R_i$ for some $i \in G$;
- $(s_1 + \cdots + s_n, s) \in R_{G,j}$ iff for every $1 \leq m \leq n$ there is $i \in G$ s.t. $(s_m, s) \in R_{i,j}$.

- Agent i is informed at least as much as agent j ($i \preceq j$) if and only if i has access to every state accessible to j.

Language

$$
\begin{aligned}
s : \phi_j \quad &::= \quad s : p_j \mid s : (\neg\phi)_j \mid s : (\phi \wedge \phi)_j \mid s : (\phi \vee \phi)_j \mid s : (\phi \rightarrow \phi)_j \\
&\qquad s : \Diamond\phi_j \mid s : BI_j(\phi_i) \mid s : DBI_j(\phi_G) \mid s : I_j(\phi_i)
\end{aligned}
$$

$$
\mathfrak{r} \quad ::= \quad R_j(s, s') \mid R_{i,j}(s, s')
$$

Informational truth-tables

$\wedge$	1	0	$*$
1	1	0	$*$
0	0	0	0
$*$	$*$	0	$*$,0

$\vee$	1	0	$*$
1	1	1	1
0	1	0	$*$
$*$	1	$*$	$*$,1

$\rightarrow$	1	0	$*$
1	1	0	$*$
0	1	1	1
$*$	1	$*$	$*$,1

$\neg$	
1	0
0	1
$*$	$*$

Satisfiability relations

1. $\mathcal{M} \Vdash R_i(s, s')$ iff $(s, s') \in R_i$
2. $\mathcal{M} \Vdash R_{i,j}(s, s')$ iff $(s, s') \in R_{i,j}$

3. $\mathcal{M} \Vdash^0 s : p_i$ iff $(p, 1) \in v(s)$ and $\mathcal{M} \Vdash R_i(s, s)$

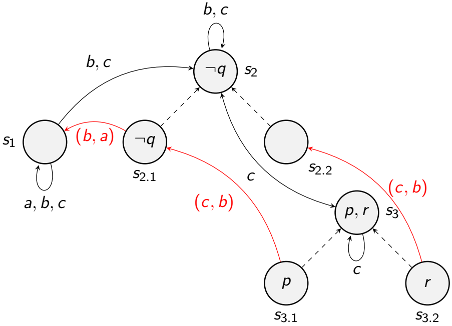
4. $\mathcal{M} \Vdash^k s : \Diamond\phi_j$ iff $\mathcal{M} \Vdash^k s' : \phi_j$ for some s' s.t. $\mathcal{M} \Vdash R_j(s, s')$
5. $\mathcal{M} \Vdash^{k+1} s : BI_j\phi_i$ iff $\mathcal{M} \Vdash^k s' : \phi_i$ for some s' s.t. $\mathcal{M} \Vdash R_{i,j}(s', s)$
6. $\mathcal{M} \Vdash^{k+1} s : DBI_j\phi_G$ iff $\mathcal{M} \Vdash^k s' : \phi_G$ for some s' s.t. $\mathcal{M} \Vdash R_{G,j}(s', s)$
7. $\mathcal{M} \Vdash^{k+1} s : I_j\phi_i$ iff $\mathcal{M} \Vdash^{k+1} s : BI_j\phi_i$ for all (at least one) $i \prec j$

8. $\mathcal{M} \Vdash^{k+1} s : I_j\phi_i$ implies $\mathcal{M} \Vdash^{k+1} s : \phi_j$

Example

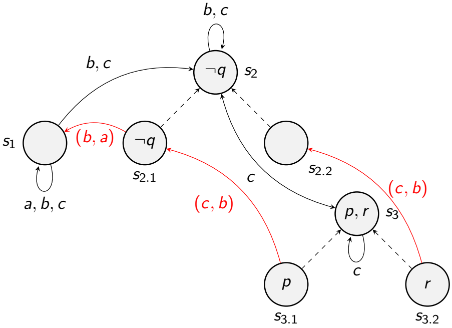
- Charles decides to share p and r stored on s_3 with Bob who can read these contents from its access to s_2. As no one in the hierarchy above Bob disagrees on those formulae, he writes them on different parts of s_2 based on whether he wants to share or not those contents.

Example



$$\mathcal{M} \Vdash R_{c,b}(s_{3.1}, s_{2.1})$$
$$\mathcal{M} \Vdash^0 s_{3.1} : p_c$$
$$\mathcal{M} \Vdash^1 s_{2.1} : BI_b p_c$$
$$\mathcal{M} \Vdash^1 s_{2.1} : I_b p_i$$
$$\mathcal{M} \Vdash^1 s_{2.1} : p_b$$
$$\mathcal{M} \Vdash R_{c,b}(s_{3.2}, s_{2.2})$$
$$\mathcal{M} \Vdash^0 s_{3.2} : r_c$$
$$\mathcal{M} \Vdash^1 s_{2.2} : BI_b r_c$$
$$\mathcal{M} \Vdash^1 s_{2.2} : I_b r_i$$
$$\mathcal{M} \Vdash^1 s_{2.2} : r_b$$

Example



$\mathcal{M} \Vdash^1 s_{2.1} p_b$

$\mathcal{M} \Vdash R_{b,a}(s_{2.1}, s_1)$

$\mathcal{M} \Vdash^2 s_1 : BI_a p_b$

$\mathcal{M} \nVdash R_{c,a}(s_{3.1}, s_1)$

$\mathcal{M} \nVdash^2 s_1 : I_a p_i$

$\mathcal{M} \nVdash^2 s_1 : p_a$

Bibliography

Cignarale, G. and Primiero, G. (2020). A multi-agent depth bounded boolean logic. In *International Conference on Software Engineering and Formal Methods*, pages 176–191. Springer.

D'Agostino, M. (2015). An informational view of classical logic. *Theoretical Computer Science*, 606:79–97.